Streaming Algorithms: Data without a disk

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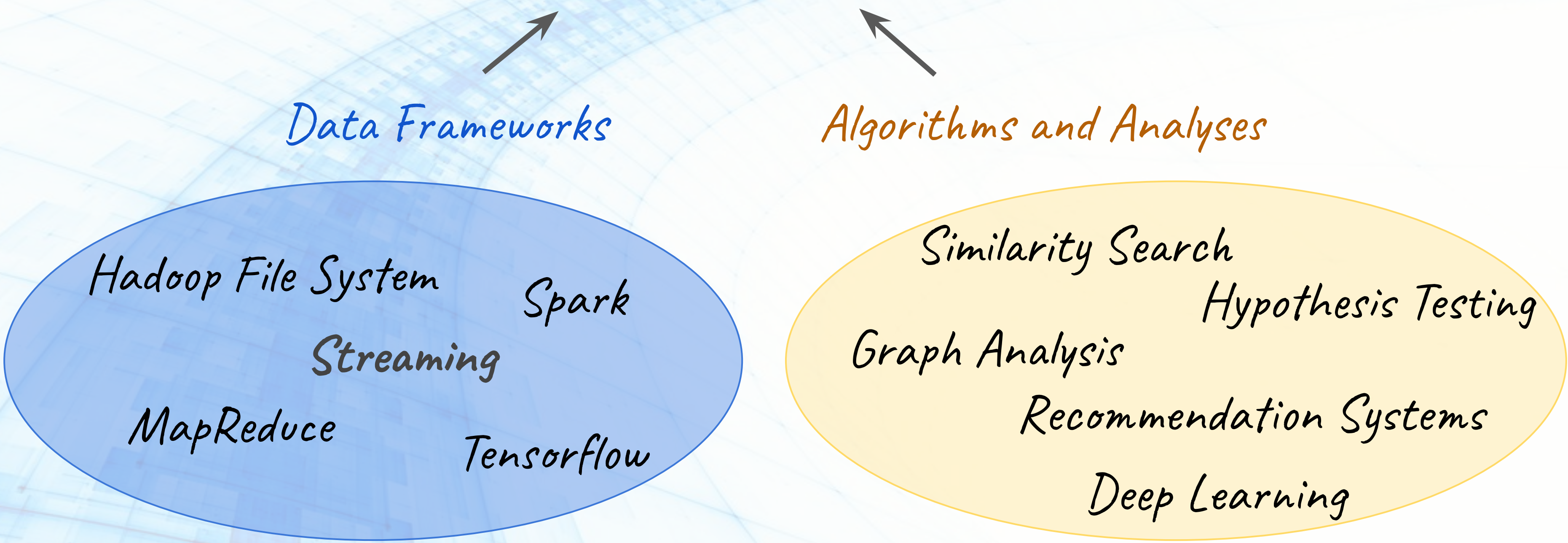
CSE545
Spring 2020

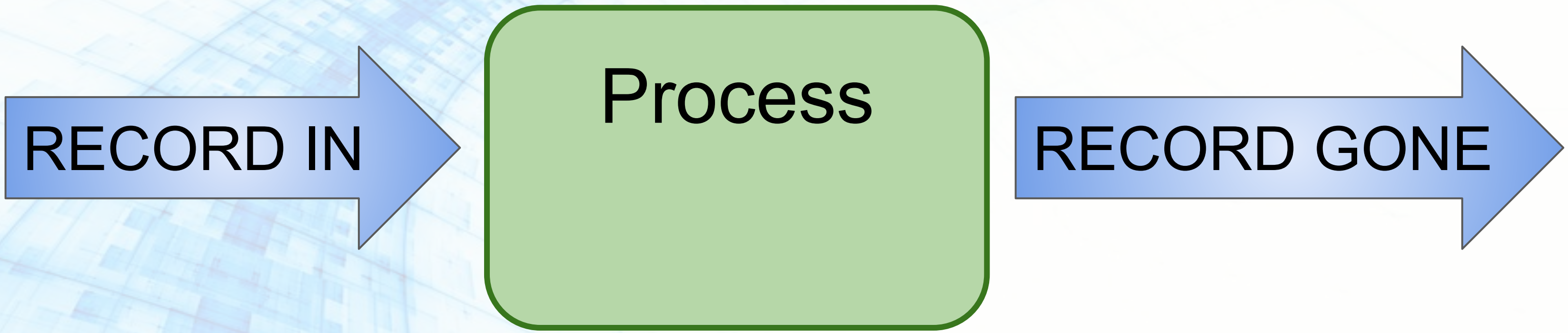
Big Data Analytics, The Class

Goal: Generalizations
A *model* or *summarization* of the data.



What is Streaming?

Broadly:



Why Streaming?

(1) **Direct:** Often, data …

- … cannot be stored (too big, privacy concerns)
- … are not practical to access repeatedly (reading is too long)
- … are rapidly arriving (need rapidly updated "results")

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Examples: *Google search queries*

Satellite imagery data

Text Messages, Status updates

Click Streams

Why Streaming?

(1) **Direct:** Often, data …

- … cannot be stored (too big, privacy concerns)
- … are not practical to access repeatedly (reading is too long)
- … are rapidly arriving (need rapidly updated "results")

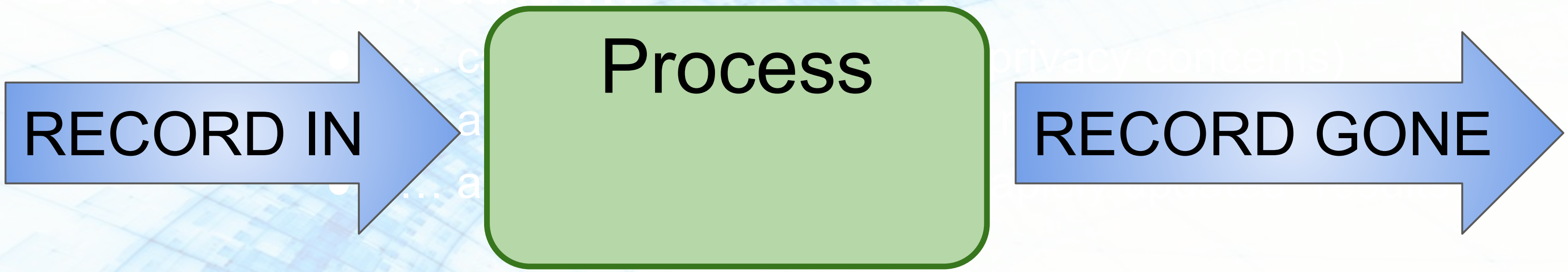
(2) **Indirect:** The constraints for streaming data force one to solutions that are often efficient even when storing data.

Streaming Approx Random Sample

Distributed IO (MapReduce, Spark)

Why Streaming?

Often translates into *O(N)* or strictly *N* algorithms.



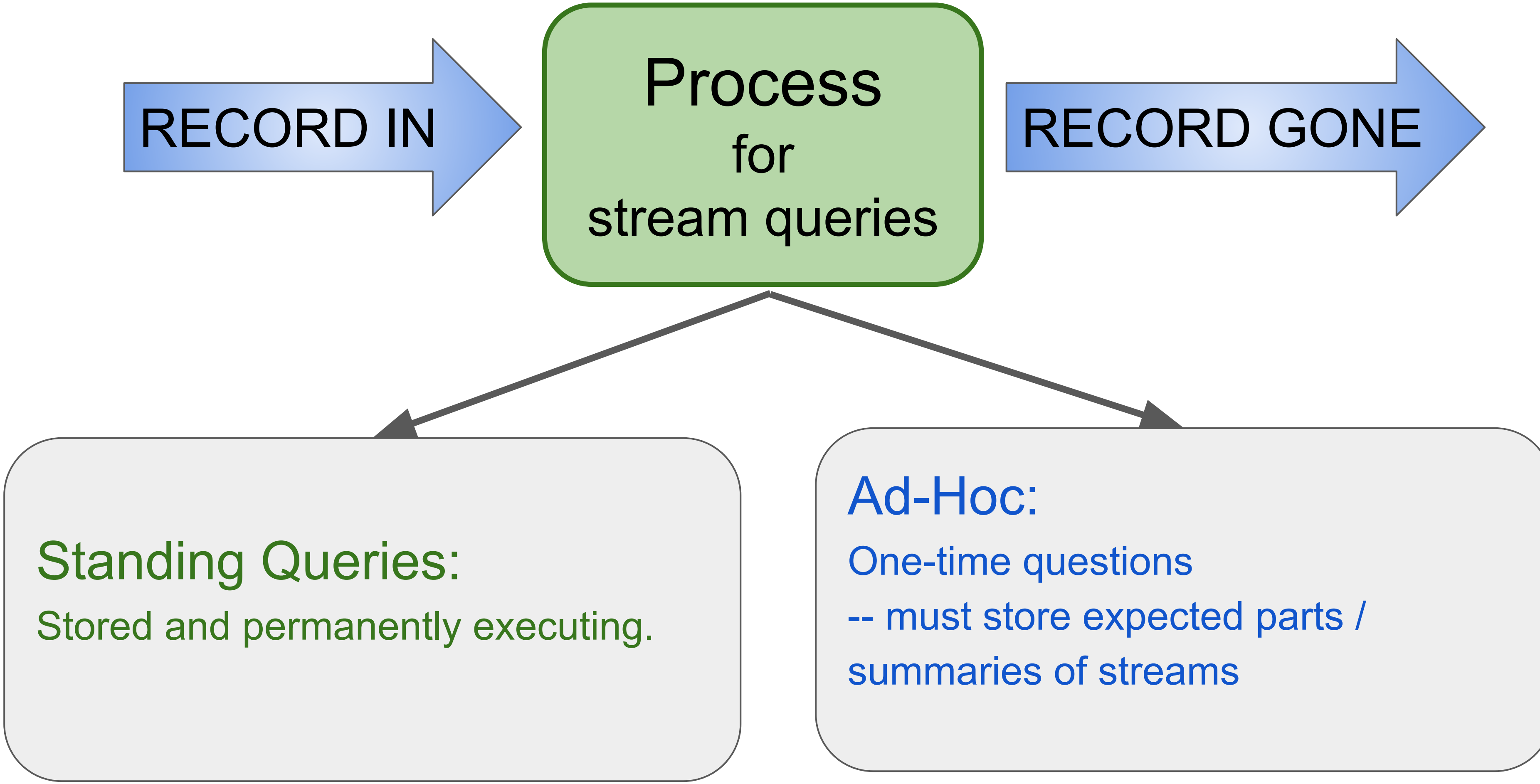
(2) **Indirect:** The constraints for streaming data force one to solutions that are often efficient even when storing data.

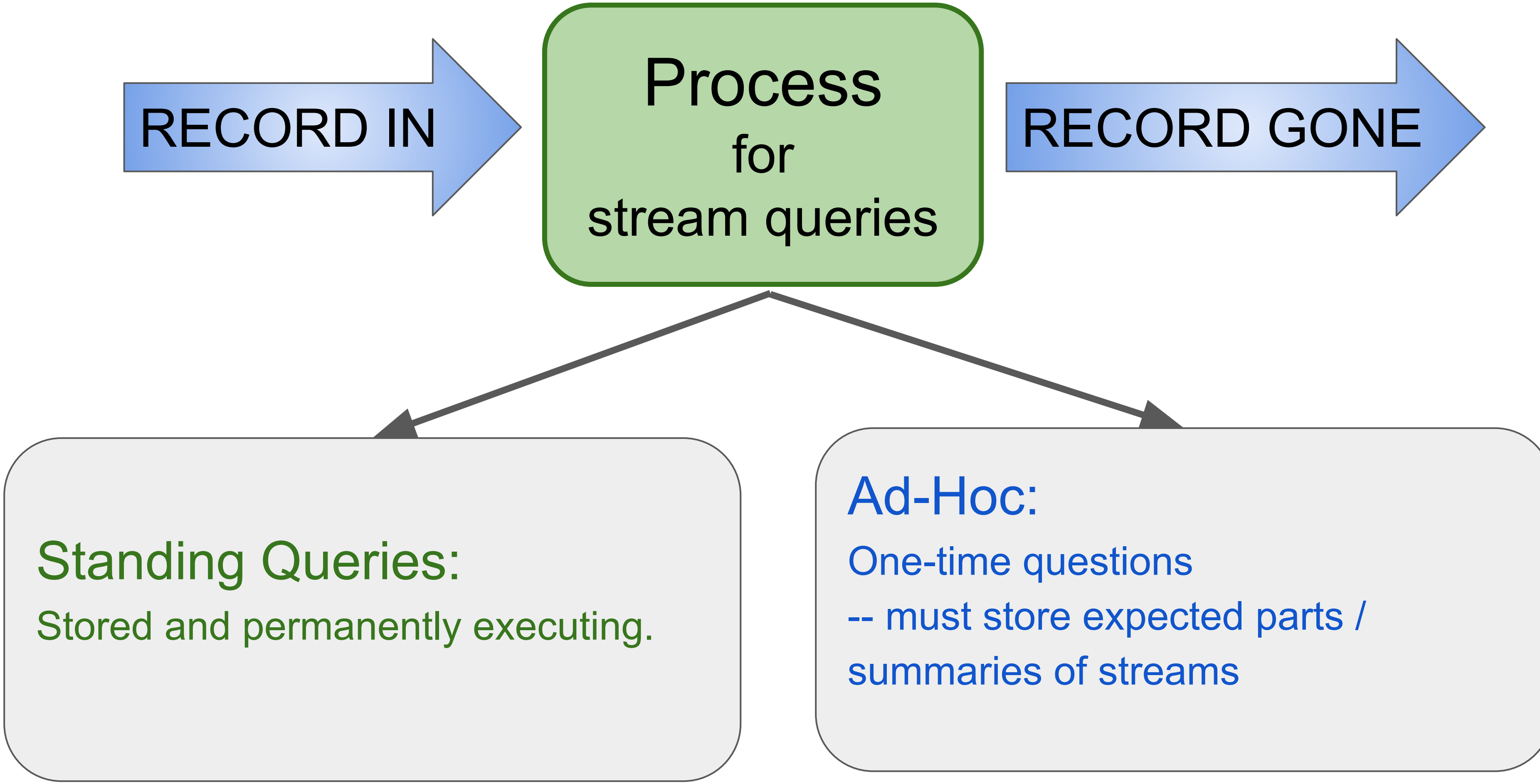
Streaming Approx Random Sample

Distributed IO (MapReduce, Spark)

Streaming Topics

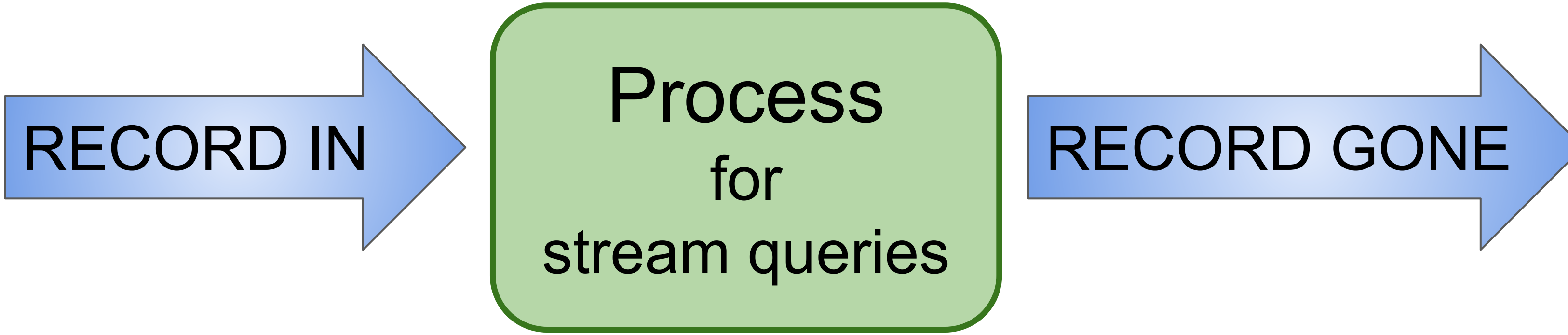
- General Stream Processing Model
- Sampling
- Counting Distinct Elements
- Filtering data according to a criteria


RECORD IN
Process
for
stream queries
RECORD GONE
Standing Queries:
Stored and permanently executing.
Ad-Hoc:
One-time questions
-- must store expected parts /
summaries of streams



E.g. How would you handle:

What is the mean of values seen so far?



Important difference from typical database management:

- Input is not controlled by system staff.
- Input timing/rate is often unknown, controlled by users.

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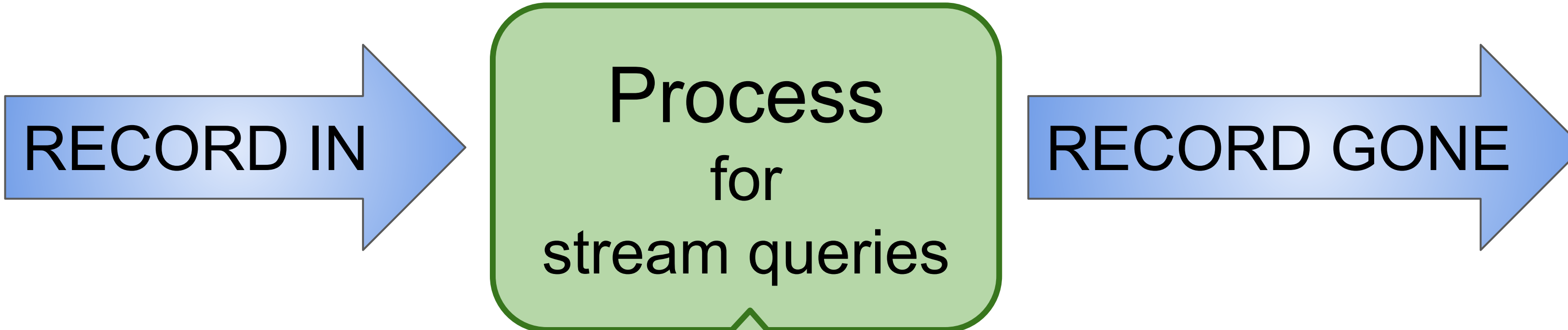

Important differen… …nagement:

- Input is n…
- Input timing/rate is … controlled by users.

Might hold a sliding window of records instead of single record.

.. , i, h, g, $\boxed{\text{f, e, d, c,}}$ b, a

E.g. How would you handle:

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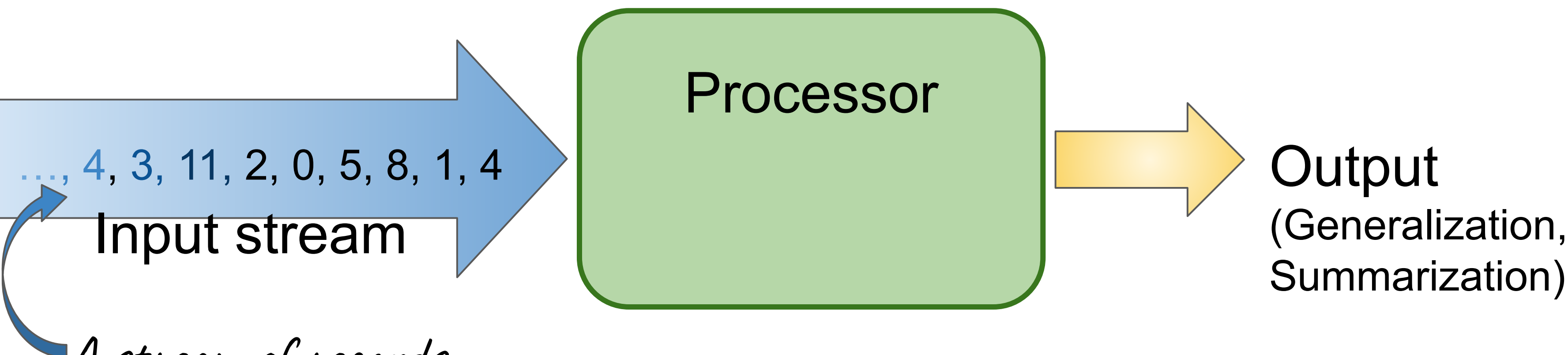
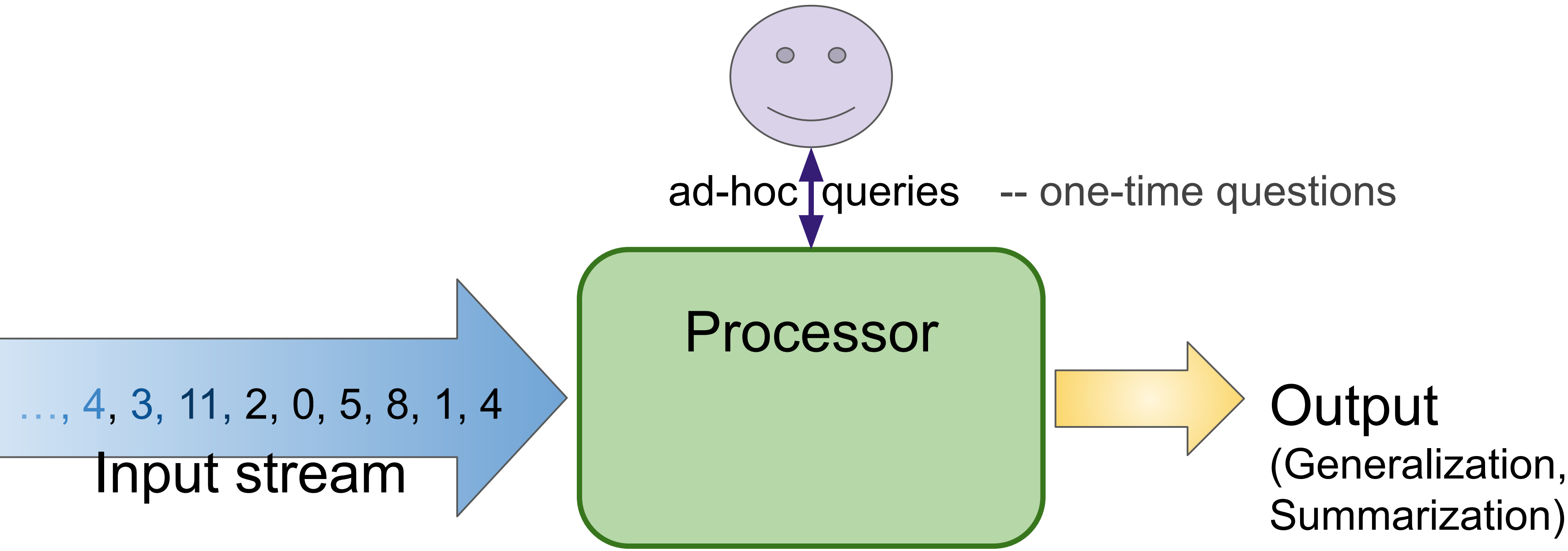
General Stream Processing Model

(Leskovec et al., 2014)



A stream of records
(also often referred to as "elements", "tuples", "lines", or "rows")
Theoretically, could be anything! search queries, numbers, bits, image files, ...

General Stream Processing Model



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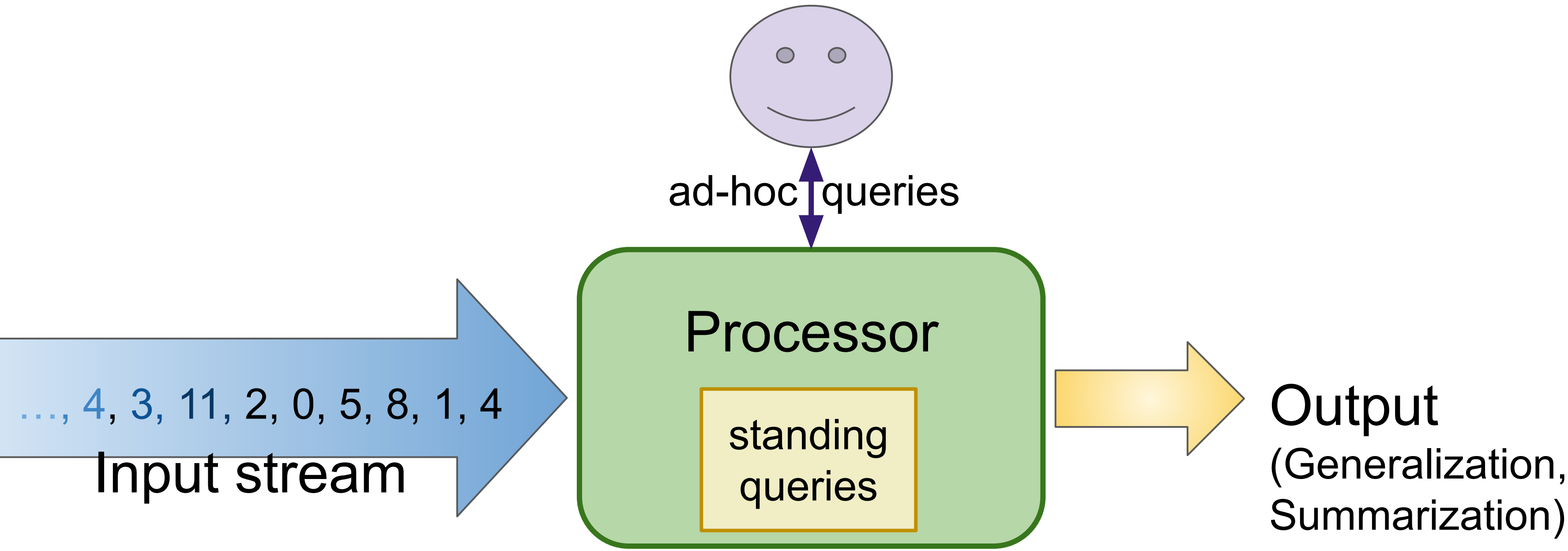
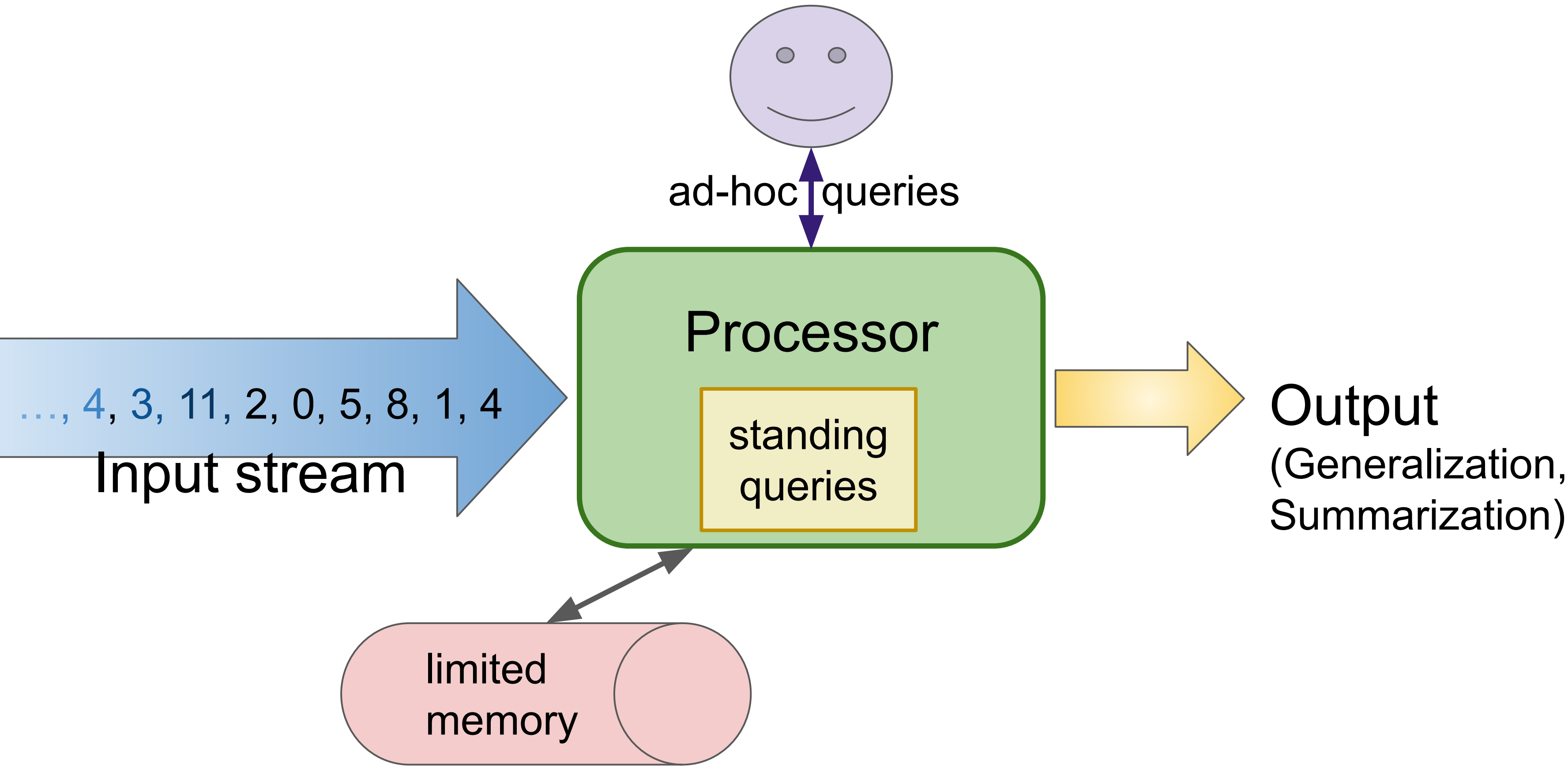


-- asked at all times.

General Stream Processing Model



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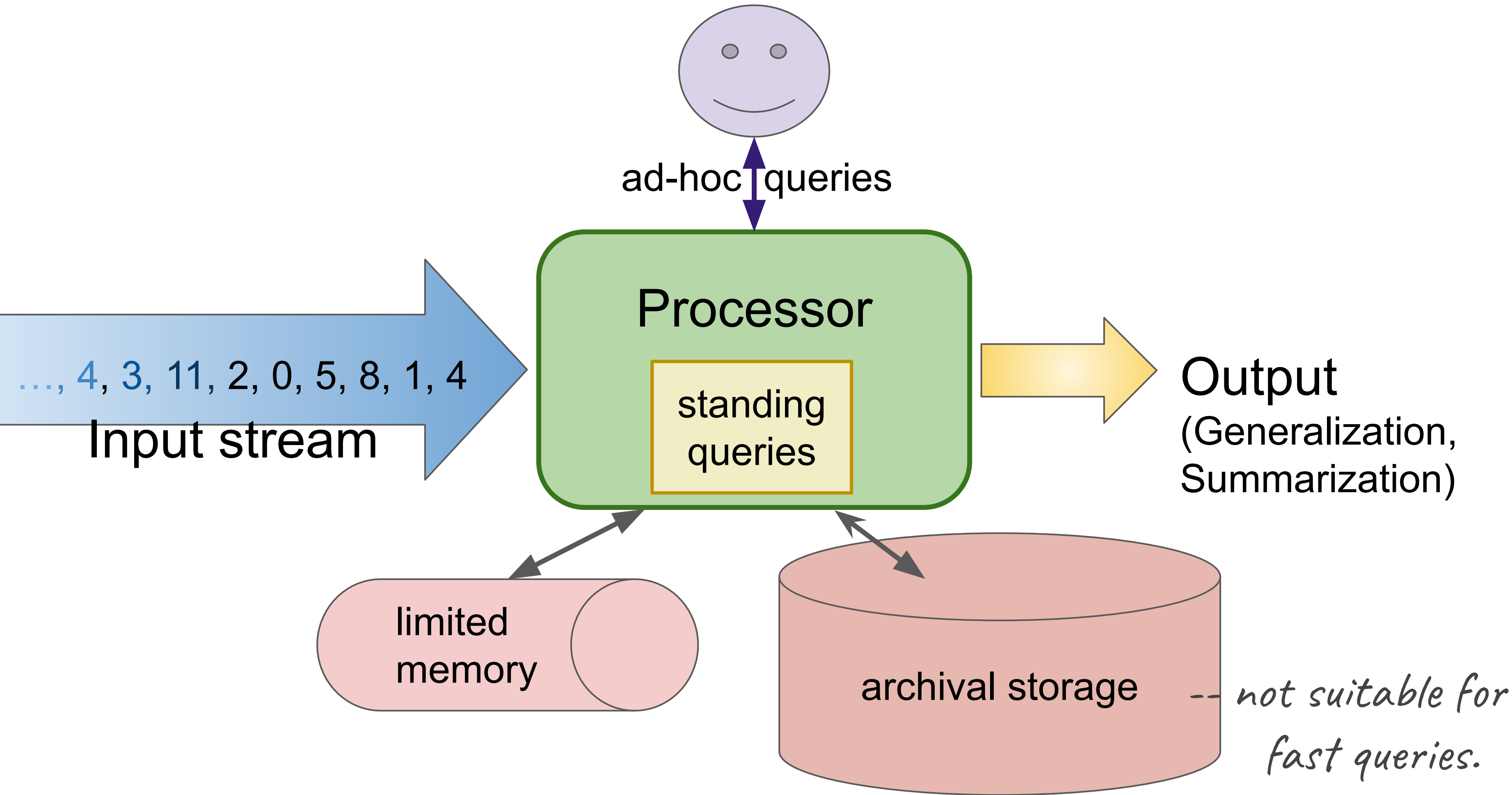

Sampling

Create a random sample for statistical analysis.

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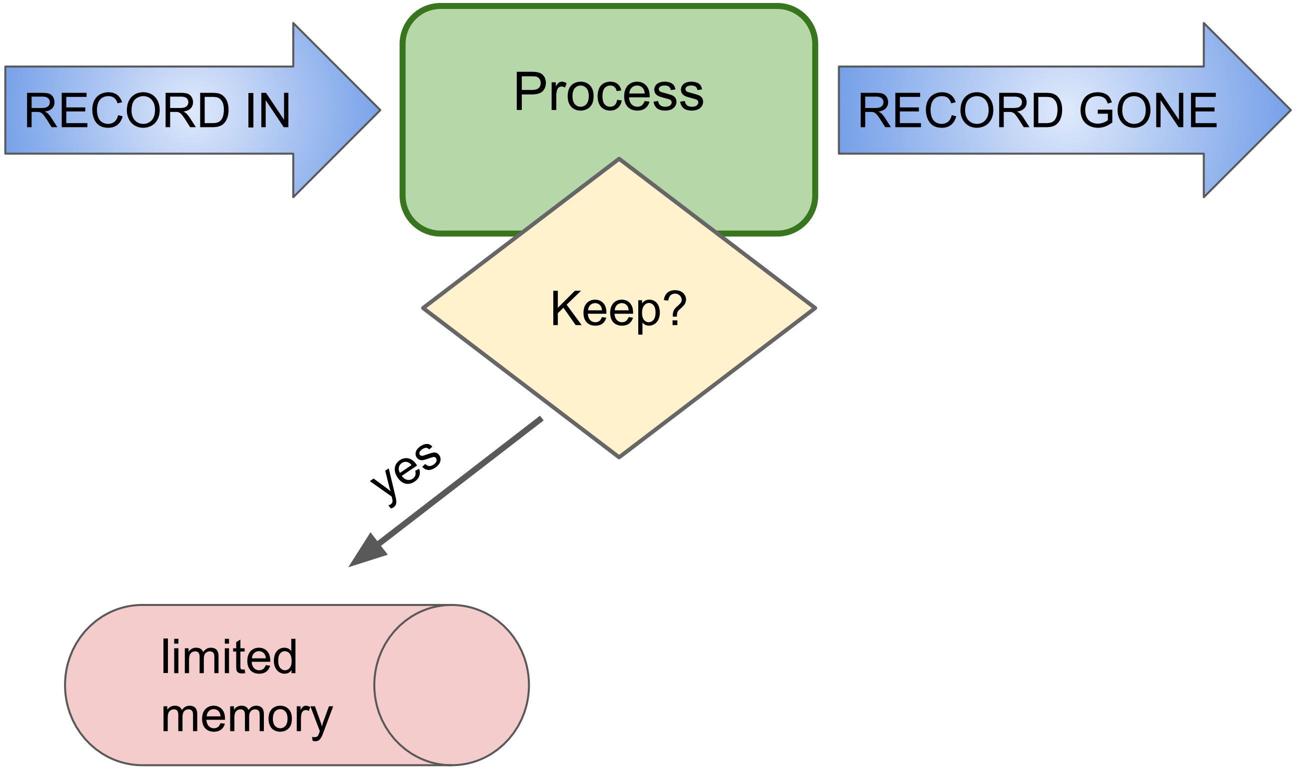

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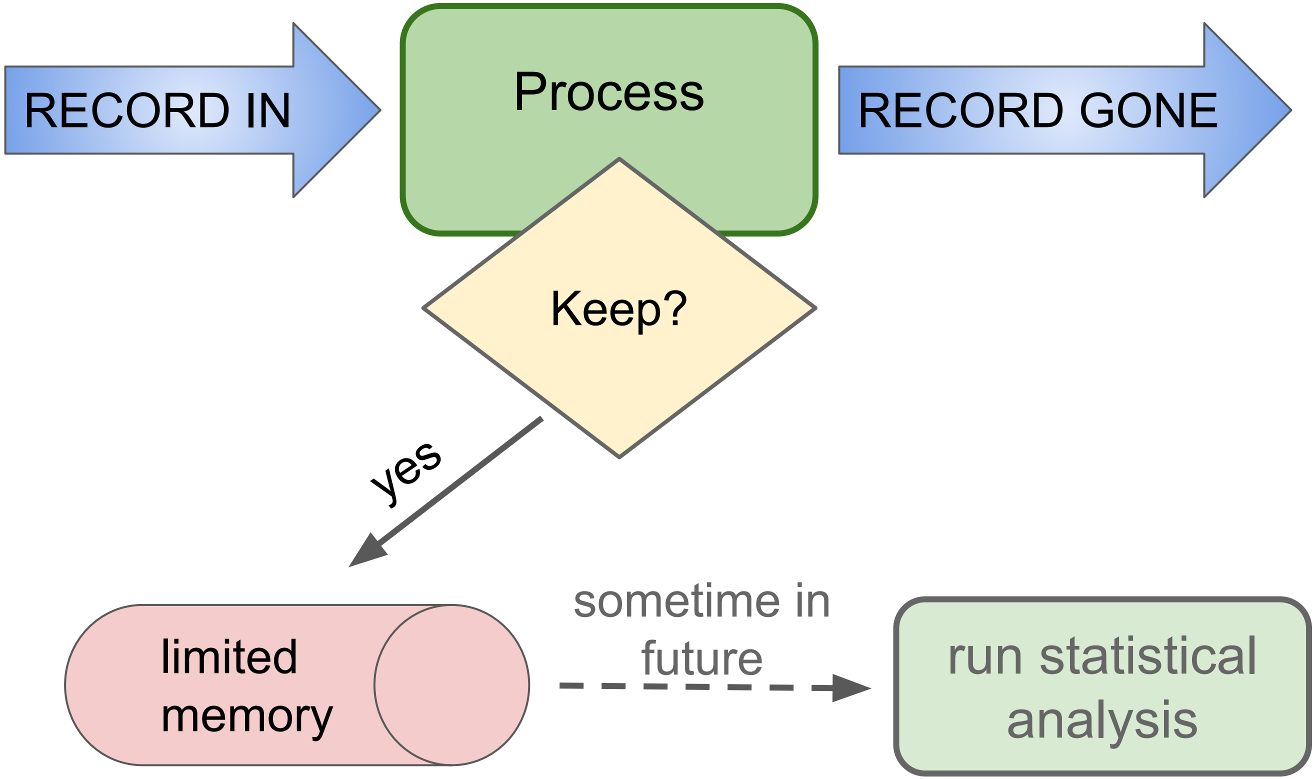

Sampling: 2 Versions

Create a random sample for statistical analysis.

1. **Simple Sampling:** Individual records are what you wish to sample.

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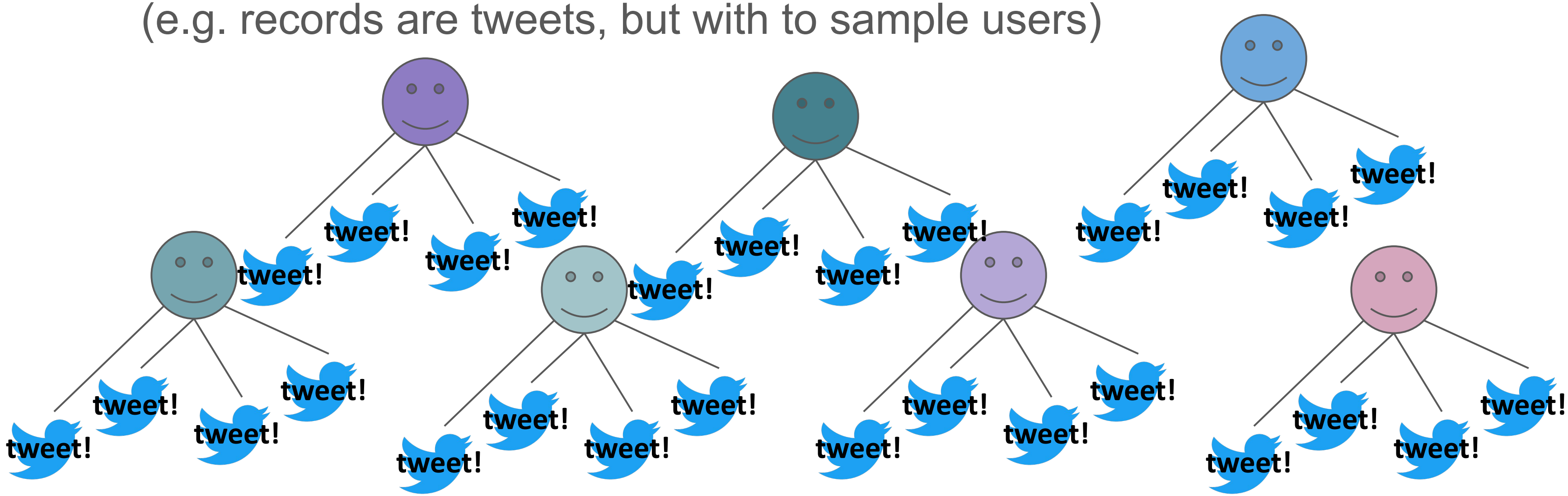
2. **Hierarchical Sampling:** Sample an attribute of a record.

   (e.g. records are tweets, but with to sample users)



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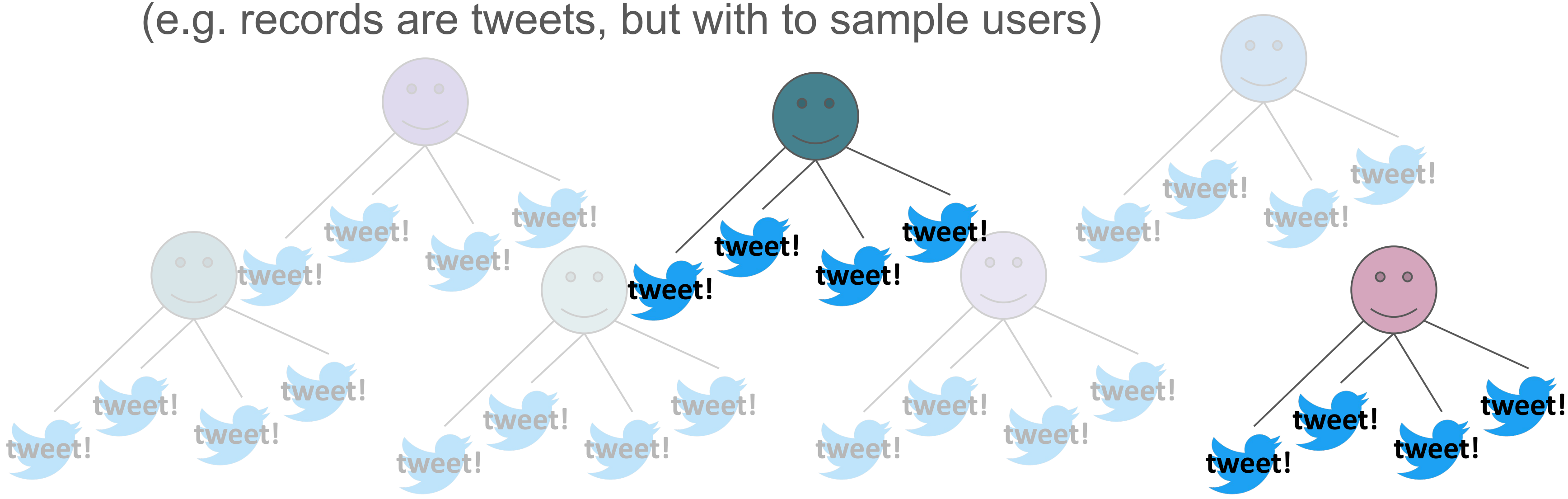

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if ?: #keep: e.g., true 5% of the time
    memory.write(record)
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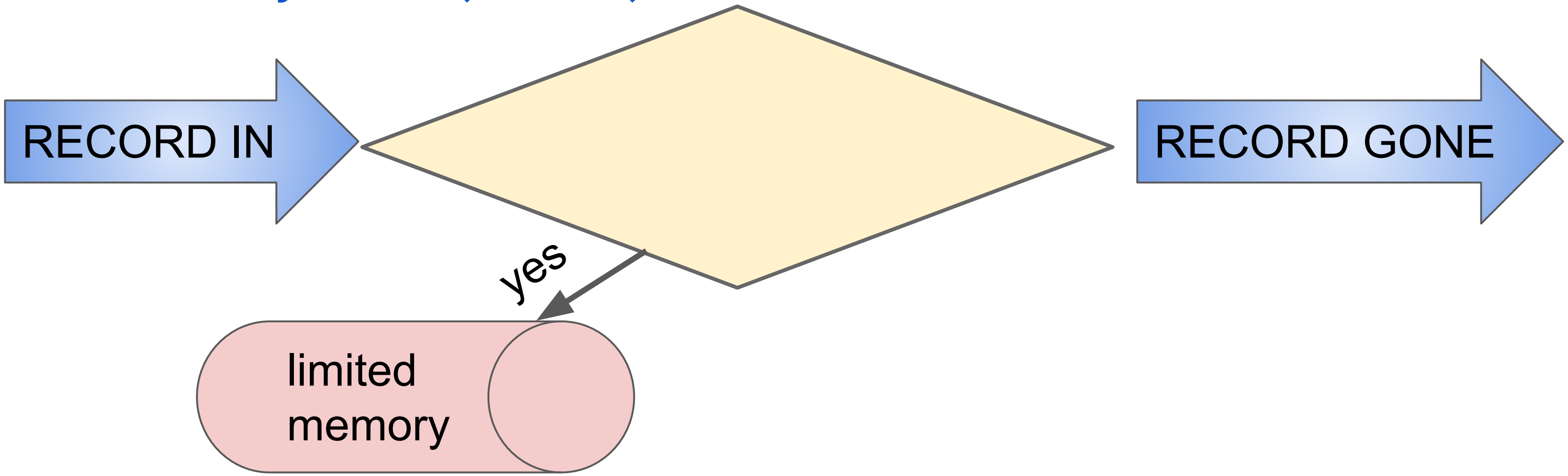

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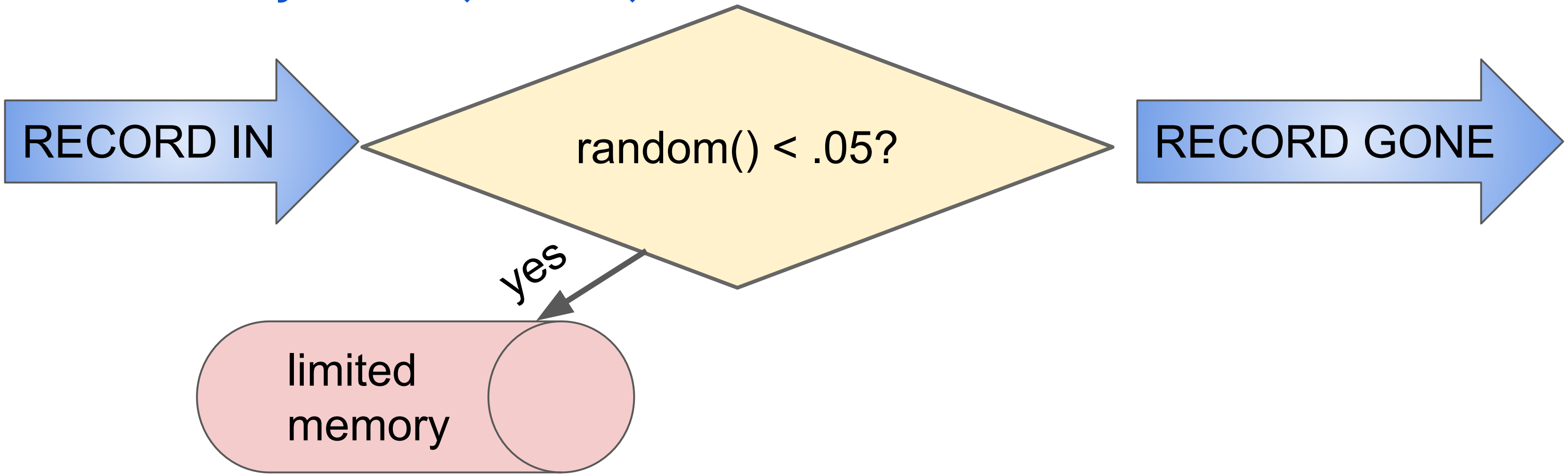

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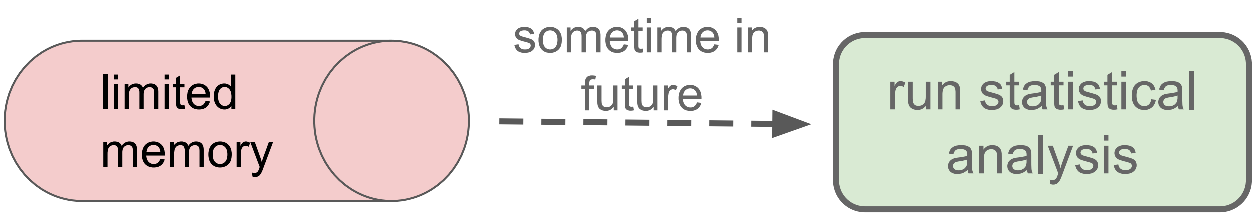
Problem: records/rows often are not units-of-analysis for statistical analyses

E.g. user_ids for searches, tweets; location_ids for satellite images



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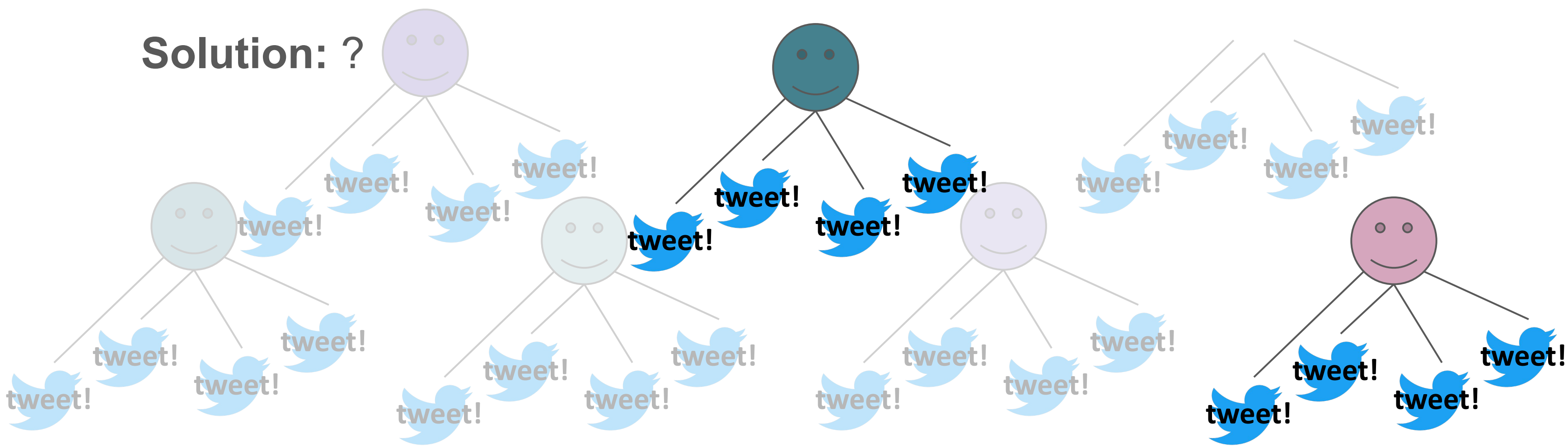

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Solution: instead of checking random digit; hash the attribute being sampled.

– streaming: only need to store hash functions; may be part of standing query

Sampling

2. **Hierarchical Sampling:** Sample an attribute of a record.

   (e.g. records are tweets, but with to sample users)

```
record = stream.next()
if hash(record['user_id']) == 1: #keep
    memory.write(record)
```

Solution: instead of checking random digit; hash the attribute being sampled.

– streaming: only need to store hash functions; may be part of standing query

How many buckets to hash into?

Streaming Topics

- General Stream Processing Model
- Sampling
- Counting Distinct Elements
- Filtering data according to a criteria

Counting Moments

Moments:

- Suppose m_i is the count of distinct element i in the data
- The kth moment of the stream is $\sum_{i \in \mathrm{Set}} m_i^k$

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- 0th moment: count of distinct elements
- 1st moment: length of stream
- 2nd moment: sum of squares
  (measures *uneveness;* related to variance)

Counting Moments

Moments:

- Suppose m_i is the count of distinct element i in the data
- The kth m… m is $\sum_{i \in \text{Set}} m_i^k$

Trivial: just increment a counter

- 0th momen… unt of distinct elements
- **1st moment: length of stream**
- 2nd moment: sum of squares (measures *uneveness;* related to variance)

Counting Moments

Applications
Counting…
distinct words in large document.
distinct websites (URLs).
users that visit a site without storing.
unique queries to Alexa.

0th moment

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Counting Momen

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0th moment
One Solution: Just keep a set (hashmap, dictionary, heap)

Problem: Can't maintain that many in memory; disk storage is too slow

- **0th moment: count of distinct elements**
- 1st moment: length of stream
- 2nd moment: sum of squares
  (measures *uneveness;* related to variance)

Counting Moments

0th moment

Streaming Solution: Flajolet-Martin Algorithm

General idea:

n -- suspected total number of elements observed

pick a hash, *h,* to map each element to $\log_2 n$ bits (buckets)

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-----------------------------------
R = 0 #current max number of zeros at tail
for each stream element, e:
    r(e) = trailZeros(h(e)) #num of trailing 0s from h(e)
    R = r(e) if r[e] > R

estimated_distinct_elements = 2^R
```

- 2nd moment: sum of squares
  (measures *uneveness;* related to variance)

Counting Momen

Mathematical Intuition

P(`trailZeros(`*h(e)*`)` >= *i*) = 2^{-i}

 # P(h(e) == __0) = .5; P(h(e) == __00) = .25; …

P(`trailZeros(`*h(e)*`)` < *i*) = 1 - 2^{-i}

for m elements: = $(1 - 2^{-i})^m$

P(one *e* has trailZeros > i) = 1 - $(1 - 2^{-i})^m$

≈ 1 - $e^{-m2^{\wedge}-i}$

If 2^R >> m, then $1 - (1 - 2^{-i})^m \approx 0$

If 2^R << m, then $1 - (1 - 2^{-i})^m \approx 1$

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$P(\texttt{trailZeros}(h(e)) < i) = 1 - 2^{-i}$

for m elements: $= (1 - 2^{-i})^m$

P(one *e* has trailZeros > i) $= 1 - (1 - 2^{-i})^m$

$\approx 1 - e^{-m2\text{^}-i}$

If $2^R >> m$, then $1 - (1 - 2^{-i})^m \approx 0$

If $2^R << m$, then $1 - (1 - 2^{-i})^m \approx 1$

Problem:

Unstable in practice.

Solution:

Multiple hash functions

but how to combine?

0th moment

Streaming Solution: Flajolet-Martin Algorithm

General idea:

n -- suspected total number of elements

pick a hash, *h*, to map each element to l

```
--------------------------------
Rs = list()
for h in hashes:
    R = 0 #potential max number of zeros at tail
    for each stream element, e:
        r(e) = trailZeros(h(e)) #num of trailing 0s from h(e)
        R = r(e) if r[e] > R
    Rs.append(2^R)

groupRs = [Rs[i:i+log n] for i in range(0, len(Rs), log n)]

estimated_distinct_elements = median(map(mean, groupRs))
```

Problem:

Unstable in practice.

Solution: Multiple hash functions

1. Partition into groups of size log n
2. Take mean in groups
3. Take median of group means

0th moment

Streaming Solution: Flajolet-Martin Algorithm

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A good approach anytime one has many “low resolution” estimates of a true value.

Counting Moments

2nd moment
Streaming Solution: Alon-Matias-Szegedy Algorithm

(Exercise; Out of Scope; see in MMDS)

- 0th moment: count of distinct elements
- 1st moment: length of stream
- **2nd moment: sum of squares (measures *uneveness* related to variance)**

Counting Moments

standard deviation

(variance squared for numeric data)

$$s = \frac{1}{N}\sqrt{\sum_{1}^{N}(x_i - \bar{x})^2}$$

Counting Moments

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$$s = \frac{1}{N}\sqrt{\sum_{1}^{N}(x_i - \bar{x})^2} = \sqrt{(\bar{x^2}) - \bar{x}^2} = \sqrt{\frac{\sum x^2}{N} - \left(\frac{\sum x}{N}\right)^2}$$

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For streaming, just need to store (1) number of elements, (2) sum of elements, and (3) sum of squares.

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However, challenge:
Sum of squares can blow up!

For streaming, just need to store (1) number of elements, (2) sum of elements, and (3) sum of squares.

Filtering Data

Filtering: Select elements with property x

Example: 40B safe email addresses for spam filter

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The Bloom Filter (approximates; allows *false positives but not false negatives*)

Given:

|S| keys to filter; will be mapped to |B| bits

hashes = $h_1, h_2, \ldots, h_k$ independent hash functions

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Algorithm:

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set all B to 0  #B is a bit vector
for each i in hashes, for each s in S:
  set B[h_i(s)] = 1 #all bits resulting from
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  ... #usually embedded in other code
while key x arrives next in stream #filter:
  if B[h_i(x)] == 1 for all i in hashes:
    #do as if x is in S
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Setup filter

Apply Filter

Filtering Data

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What is the probability of a *false positive (FP)*?

Q: What fraction of |B| are 1s?

(Leskovec et al., 2014)

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Q: What fraction of |B| are 1s?

A: Analogy:

Throw |S| * k darts at n targets.

1 dart: $1/n$

d darts: $(1 - 1/n)^d$ = prob of 0

= $e^{-d/n}$ are **0s**

(Leskovec et al., 2014)

Filtering Data

Filtering: Select elements with property x

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The Bloom Filter (approximates; allows *FPs*)

Given:

$|S|$ keys to filter; will be mapped to $|B|$ bits

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$= e^{-1}$

for large n

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thus, $(1 - e^{-d/n})$ are **1s**

probability all k being 1?

(Leskovec et al., 2014)

Filtering Data

Filtering: Select elements with property x

Example: 40B safe email addresses for spam filter

The Bloom Filter (approximates; allows *FPs*)

Given:

|S| keys to filter; will be mapped to |B| bits

hashes = $h_1, h_2, \ldots, h_k$ independent hash functions

Algorithm:

```
set all B to 0
for each i in hashes, for each s in S:
  set B[h_i(s)] = 1
  … #usually embedded in other code
while key x arrives next in stream #filter:
  if B[h_i(x)] == 1 for all i in hashes:
    #do as if x is in S
  else: #do as if x not in S
```

What is the probability of a *false positive*?

Q: What fraction of |B| are 1s?

A: Analogy:
Throw |S| * *k* darts at *n* targets.
1 dart: $1/n$
d darts: $(1 - 1/n)^d$ = prob of 0
$= e^{-d/n}$ are **0s**

thus, $(1 - e^{-d/n})$ are **1s**

probability all *k* being 1?
$\mathbf{(1 - e^{-(|S|*k)/n})^k}$

Note: Can expand S as stream continues as long as |B| has room (e.g. adding verified email addresses)

(Leskovec et al., 2014)

Streaming Topics

- General Stream Processing Model
- Sampling
  - approx. random
  - hierarchical approx. random
- Counting Elements
  - distinct elements
  - mean, standard deviation
- Filtering data according to a criteria
  - bloom filter setup + application
  - calculating false positives